

# Reduced-Dimensionality Geometric Approach to Fault Identification in Stochastic Structural Systems

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An information-based reduced-dimensionality formulation of the recently introduced geometric approach to fault identification in stochastic structural systems is presented. The feature vector, which conveys the system information used in fault identification and is a key element of the general geometric approach, is transformed into a suitable coordinate system, within which information compression may be best achieved. Dimensionality selection is subsequently based on bounding of the information loss, expressed in terms of logarithmic entropy, below a certain threshold. The information-based reduced-dimensionality formulation thus presents a formal and systematic procedure for the proper selection of a minimal dimensionality feature vector and leads to substantial simplification and potentially improved fault identification. Its application to fault identification in a laboratory-scale beam and a planar truss structure demonstrates high effectiveness and performance characteristics essentially equivalent to those of the full-dimensionality formulation.

## Nomenclature

$F_\alpha^i$	= fault of magnitude $\alpha$ within the $i$ th fault mode
$g^i(\theta_K) = 0$	= equation of the $i$ th fault mode hyperplane
$H_R(s)$	= entropy function associated with the vector $s$
$\bar{H}_R(s_M, s)$	= ratio of entropy functions of the vectors $s_M$ and $s$
$M$	= dimension of $s_M$ ( $M < N$ )
$N$	= dimension of the original feature vector
$N_F$	= number of fault modes
$P_s$	= transformed feature vector covariance ( $N \times N$ )
$P_{s_M}$	= reduced transformed feature vector covariance ( $M \times M$ )
$P_\theta$	= feature vector covariance ( $N \times N$ )
$p$	= number of faults per fault mode considered during training
$s$	= transformed feature vector ( $N \times 1$ )
$s_M$	= reduced transformed feature vector ( $M \times 1$ )
$T$	= feature vector transformation matrix ( $M \times N$ )
$\theta$	= feature vector composed of the model parameters ( $N \times 1$ )
$\theta_K$	= vector of the first- or second-order moments of $s_M$ ( $\rho \times 1$ )
$\mu_s$	= transformed feature vector mean ( $N \times 1$ )
$\mu_{s_M}$	= reduced transformed feature vector mean ( $M \times 1$ )
$\mu_\theta$	= feature vector mean ( $N \times 1$ )

## Superscripts

$i$	= $i$ th fault mode
$i_j$	= $j$ th fault within the $i$ th fault mode
$u$	= unclassified current fault

## I. Introduction

**F**AULT detection and identification in structural systems, such as aerospace and mechanical structures, buildings, bridges, and offshore platforms, is of paramount importance for reasons associated with safety and proper maintenance. Techniques aiming at

nondestructive detection and identification of structural faults, based on the study of behavioral discrepancies between the nominal (unfailed) and failed systems, thus have received considerable attention in recent years.<sup>1-5</sup>

A large category of structural fault identification methods is based on the evaluation of changes incurred in the system's structural stiffness matrix,<sup>6</sup> usually under the constant mass matrix assumption. These methods may use static test data,<sup>7</sup> dynamic test data,<sup>1,8,9</sup> or combinations of the two.<sup>3</sup> The main idea is in the use of the test data to formulate an optimization problem through which the changes of the structural stiffness matrix from its nominal (unfailed) value (often determined from a finite element model of the structure) are computed. The methods achieve simultaneous fault detection and identification based on the determination of the changed stiffness matrix elements.

An alternative category of methods is based on the evaluation of changes incurred in the structural system's modal parameters.<sup>4,10,11</sup> The basic idea is the comparison of the modal parameters of the nominal system to those of the failed system and the detection and identification of faults through observation of the incurred change patterns.

Both categories of methods, nevertheless, are characterized by a number of drawbacks and limitations: Static testing may be a problem with certain structures, whereas complete eigenmode information (required in the dynamic test case) is difficult to obtain and requires elaborate testing procedures. The determination of eigenmode information additionally requires cumbersome numerical procedures for the solution of nonlinear equations and in any case, high-frequency eigenmodes are difficult to obtain but their omission may seriously affect effectiveness. Estimation of a complete structural model, which may be of considerably high dimensionality, is also necessary, and this leads to a large nonlinear optimization problem. In addition, stochastic noise effects are not always accounted for, an omission that may lead to further difficulties.<sup>9</sup> The fact that the eigenmode estimates are themselves stochastic quantities with inherent variability is not taken into account either, with the result being lack of sufficient certainty that an observed change in any given parameter is due to a fault and not to the inherent variability of the test data.<sup>11</sup>

In addition, the first category of methods may lead to problematic fault localization, as a particular stiffness matrix element may admit contributions from several structural components sharing the same node. The second category, on the other hand, relaxes the requirement for stiffness matrix determination, but properly identifying

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faults from mode shape pattern changes may not always be possible.

In a recent paper by the authors,<sup>12</sup> a novel geometric approach to the nondestructive identification of faults in stochastic structural systems was introduced. This approach is distinctly different from previous schemes, making use of reduced-order and partial stochastic structural models, a properly selected feature vector, a corresponding feature space equipped with a proper metric, and the notion of fault mode as the union of faults of all possible magnitudes but of common cause. (All fault modes of interest should be properly defined in order to be amenable to identification.)

The geometric approach is based on the fact that fault-induced changes in the structural system's properties affect the selected model parameters, and, through them, the feature vector statistical characteristics. Within the selected feature space, all individual (fixed-magnitude) faults assume a pointwise representation, whereas each fault mode is represented as a proper subspace. Approximate representations of these subspaces are preconstructed in a training stage. Once a system fault is detected on the basis of operating data, an interval estimate of the current feature vector, defining its point representation in the feature space, is obtained. Fault identification then is accomplished by determining the specific fault mode subspace within which the current fault point lies.

The geometric fault identification approach overcomes many of the limitations of the previous structural fault identification schemes because it accounts for stochastic effects and the inherent variability of test data, requires a minimal number of measurement locations, is capable of operating on any type (acceleration, velocity, or displacement) of vibration test data, requires only partial [even single input/single output (SISO)] and reduced-order models, and eliminates the difficulties associated with stiffness matrix- or modal-parameter-based procedures. In addition, it is very general and potentially capable of identifying faults within any type of linear or nonlinear structural system.

In the present paper, an information-based reduced-dimensionality formulation of the geometric fault identification approach is introduced. In contrast to the original formulation, in which the feature vector is selected arbitrarily to consist of the estimated model parameters, the present formulation uses an information-based framework for feature vector, and thus feature space, transformation and reduction. The feature vector thus is transformed into a suitable coordinate system, within which information compression, and hence feature vector reduction, may be best achieved. Feature vector dimensionality selection is then based on bounding of the information loss, expressed in terms of logarithmic entropy, below a certain threshold.

The information-based reduced-dimensionality geometric fault identification approach therefore offers a formal and systematic procedure for the proper selection of a minimal-dimensionality feature vector, and achieves two important goals: 1) substantial simplification of the fault identification procedure by way of reduced feature space dimensionality and 2) potentially improved fault identification performance over arbitrarily truncated feature vectors.

The rest of this paper is organized as follows: A brief overview of the main ideas pertaining to the geometric fault identification approach is presented in Sec. II. The information-based reduced-dimensionality formulation is presented in Sec. III, and its application to fault identification in a laboratory-scale, simply supported beam and a finite element model of a planar truss structure is discussed in Sec. IV. Finally, the conclusions are summarized in Sec. V.

## II. Overview of the Main Ideas

The geometric fault identification approach is based on vibration test data, typically force excitation along with the resulting vibration displacement, velocity, or acceleration, measured at selected locations on the structure. It consists of the following elements: 1) a partial (even SISO) and reduced-order mathematical system model, 2) a properly selected feature vector, 3) a feature space equipped with a proper metric, and 4) the notion of fault mode and its geometrical representation.

The mathematical system model is of the discrete-time stochastic dynamical type and provides a partial and reduced-order repre-

sentation of the actual structure. These features are very important because a complete system model may be not only difficult to construct but also difficult or impractical to use. The model typically is obtained through identification techniques using vibration test data.<sup>13,14</sup>

The feature vector  $\theta \in \mathcal{R}^N$  consists of selected model parameters that convey system information. The feature space is a space spanned by the first- and, potentially, second-order moments of the feature vector, collected into the vector  $\theta_K \in \mathcal{R}^p$ . The feature space also is equipped with a proper metric.

The notion of fault mode refers to the union of faults of all possible magnitudes (severities) but common cause. Within this context a particular fault from fault mode  $i$  and of magnitude  $a$  is represented as  $F_a^i$ , with the  $i$ th fault mode thus defined as  $F^i = \{F_a^i \mid a \in A \subseteq \mathcal{R}\}$  in the one-dimensional fault magnitude case.

Within the selected feature space, all individual (fixed-magnitude) faults admit a pointwise representation. A fault mode, being a continuum of variable-magnitude faults, admits, on the other hand, a subspace representation. The dimensionality of this subspace depends on the dimensionality of the fault magnitude, the accurate definition of which may or may not be possible.

The system models used in this work are of the autoregressive moving average with exogenous input (ARMAX/ARX) type.<sup>13,14</sup> The feature vector  $\theta$  consists of selected autoregressive, exogenous, and moving average parameters (see Sec. IV), whereas the vector  $\theta_K$  defining the feature space may include the mean values and covariances of the feature vector elements. The metric defined on the feature space may be of either the Euclidean or Kullback type.<sup>12</sup> An example of a fault  $F_a^i$  (of magnitude  $a$ ) may be the weakening of the elasticity characteristics of a structural element or joint (element or joint  $i$ ). It is clear that an infinite number of faults (each one corresponding to a specific magnitude) is generally possible, with their union defining a particular fault mode  $F^i$ .

The operational stages of the geometric fault identification approach are as follows:

1) In an initial training stage, and with the aid of identification techniques and measurements obtained from a detailed simulation (often finite element) model of the structure, a partial and reduced-order nominal system model is obtained. Based on this, a proper feature vector and the corresponding feature metric space are selected. Approximate geometric representations of the various fault mode subspaces are constructed by using training data (obtained by injecting faults of various magnitudes into the simulation model) and appropriate regression techniques.<sup>15</sup>

2) Once the presence of a system fault is detected on the basis of periodically obtained system (experimental) data and statistical fault detection schemes,<sup>12</sup> an interval (mean and covariance) estimate of the current feature vector, defining a point that represents the current (unknown) fault in the feature space, is computed.

3) Finally, the fault identification problem is viewed as the problem of determining the specific fault mode subspace within which the incurred fault lies.

Note that, because of estimation and modeling inaccuracies, the point representing the current fault may not strictly belong to its proper subspace, but to the latter's immediate vicinity. To account for this, the fault identification problem is formulated as a geometric minimal distance one, according to which the current fault is associated with the fault mode with the subspace of which its distance (computed via a constrained optimization scheme) is minimal.

## III. Information-Based Reduced-Dimensionality Geometric Approach

The reduced-dimensionality geometric fault identification approach is developed by using an information-based formalism that allows for the effective treatment of two main issues: 1) feature vector transformation into a coordinate system within which information compression, and thus feature vector reduction, may be best achieved; and 2) feature vector dimensionality selection.

### A. Feature Vector Transformation and Reduction

Let the feature vector  $\theta \in \mathcal{R}^N$ , composed of selected model parameters, be characterized by mean  $\mu_\theta$  and covariance  $P_\theta$ . Also let

$P \in \mathbb{R}^{N \times N}$  indicate an arbitrary (symmetric and positive-definite) covariance matrix.  $P$  may be decomposed as

$$P = U \cdot \Lambda \cdot U^T \quad (1)$$

where

$$\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N) \in \mathbb{R}^{N \times N} \quad (2)$$

$$U = [\mathbf{u}_1 \ \mathbf{u}_2 \ \dots \ \mathbf{u}_N] \in \mathbb{R}^{N \times N} \quad (3)$$

with  $\text{diag}(\dots)$  designating a diagonal matrix composed of the indicated elements,  $\lambda_i (>0)$  ( $i = 1, 2, \dots, N$ ) the  $i$ th eigenvalue of  $P$ , and  $\mathbf{u}_i$  the corresponding normalized eigenvector. The eigenvectors are thus orthonormal, satisfying the relation  $\mathbf{u}_i^T \cdot \mathbf{u}_j = \delta_{ij}$ , with  $\delta_{ij}$  denoting the Kronecker delta ( $\delta_{ij} \equiv 0$  for  $i \neq j$ ,  $\delta_{ii} \equiv 1$ ). The normalized eigenvector matrix  $U$  therefore satisfies the relation  $U \cdot U^T = I_N$  ( $I_N$  denoting the  $N$ -dimensional identity matrix). In view of this,  $\Lambda$  may be expressed as

$$\Lambda = U^T \cdot P \cdot U \quad (4)$$

The feature vector  $\theta$  then may be analyzed in terms of the orthonormal vectors  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N\}$ , which constitute a basis in  $\mathbb{R}^N$ , as

$$\theta = \sum_{j=1}^N s_j \mathbf{u}_j = [\mathbf{u}_1 \ \dots \ \mathbf{u}_N] \begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix} = U \cdot \mathbf{s} \quad (5)$$

with  $\mathbf{s} \in \mathbb{R}^N$  denoting the transformed [in the  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N\}$  coordinate system] feature vector. Premultiplying Eq. (5) by  $U^T$  yields the following expression for the transformed feature vector:

$$\mathbf{s} = U^T \cdot \theta \Leftrightarrow s_j = \mathbf{u}_j^T \cdot \theta \quad (j = 1, 2, \dots, N) \quad (6)$$

from which its mean and covariance are obtained as

$$\mu_s = U^T \cdot \mu_\theta, \quad \text{Cov}[\mathbf{s}] \triangleq P_s = U^T \cdot \text{Cov}[\theta] \cdot U \quad (7)$$

Feature vector reduction to dimensionality  $M (< N)$  may be achieved in the  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N\}$  coordinate system as follows: Let  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_M\}$  denote the eigenvectors that correspond to the largest  $M$  variances  $\text{Var}[s_j]$  ( $j = 1, 2, \dots, N$ ). Expression (5) then may be rewritten as

$$\theta = \sum_{j=1}^M s_j \mathbf{u}_j + \sum_{j=M+1}^N s_j \mathbf{u}_j = \begin{bmatrix} U_M & \vdots & U_{N-M} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{s}_M \\ \vdots \\ \mathbf{s}_{N-M} \end{bmatrix} \quad (8)$$

with  $U_M \in \mathbb{R}^{N \times M}$ ,  $U_{N-M} \in \mathbb{R}^{N \times (N-M)}$  denoting the indicated partitions (submatrices) of  $U$ , and  $\mathbf{s}_M \in \mathbb{R}^M$ ,  $\mathbf{s}_{N-M} \in \mathbb{R}^{N-M}$  the indicated partitions (subvectors) of  $\mathbf{s}$ .

Keeping only the first  $M$  coordinates  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_M\}$  in expression (8), one has

$$\theta = \sum_{j=1}^M s_j \mathbf{u}_j + \epsilon \quad (9)$$

with  $\epsilon$  indicating the error vector due to coordinate-system reduction.

Premultiplying Eq. (8) by  $U_M^T$  and using the eigenvector orthonormality property, the reduced transformed feature vector is obtained as

$$\mathbf{s}_M = U_M^T \cdot \theta \Rightarrow \mathbf{s}_M = T \cdot \theta \quad (10)$$

with  $T = U_M^T \in \mathbb{R}^{M \times N}$  representing the transformation matrix. Hence

$$\mu_{s_M} = T \cdot \mu_\theta, \quad \text{Cov}[\mathbf{s}_M] \triangleq P_{s_M} = T \cdot \text{Cov}[\theta] \cdot T^T \quad (11)$$

*Remark.* Note that for  $P$  equal to the covariance  $P_\theta$  of the feature vector  $\theta$ , Eq. (5) represents the latter's Karhunen-Loève

expansion.<sup>16</sup> In that case, because of property (4), the covariance matrix (7) of the transformed feature vector  $\mathbf{s}$  becomes

$$\text{Cov}[\mathbf{s}] = U^T \cdot \text{Cov}[\theta] \cdot U = \Lambda \quad (12)$$

This indicates that the Karhunen-Loève expansion diagonalizes the feature vector covariance matrix or, in other words, transforms the feature vector  $\theta$  into a vector with mutually uncorrelated components. It is known<sup>16</sup> that the Karhunen-Loève expansion leads to optimal, in the sense of minimum mean square error  $E\{\epsilon^T \epsilon\}$  [see Eq. (9)], information compression and thus to optimal feature vector reduction for any selected dimensionality. Furthermore, because the variances  $\text{Var}[s_j]$  are, in this case, equal to  $\lambda_j$  ( $j = 1, 2, \dots, N$ ), the coordinates  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_M\}$  are those associated with the  $M$  largest eigenvalues of  $P_\theta$ .  $\square$

Within the context of the reduced-dimensionality geometric approach, a transformation matrix  $T \in \mathbb{R}^{M \times N}$  obtained from a single covariance matrix  $P$  needs to be used for transforming the feature vector. Two possibilities are considered.

1)  $P = P_{\theta^0}$ , with  $P_{\theta^0}$  representing the feature vector covariance for the nominal (unfailed) system, or

2)  $P = P_{\theta^{\text{av}}}$ , with  $P_{\theta^{\text{av}}}$  representing an average feature covariance matrix obtained during the training stage.

Denoting as  $\theta^{ij}$  the feature vector obtained from data corresponding to the  $j$ th fault ( $j = 1, 2, \dots, p$ ) from within the  $i$ th fault mode ( $i = 1, 2, \dots, N_F$ ), an average feature covariance matrix may be obtained as

$$P_{\theta^{\text{av}}} = \frac{1}{p \cdot N_F} \sum_{i=1}^{N_F} \sum_{j=1}^p P_{\theta^{ij}} \quad (13)$$

## B. Feature Vector Dimensionality Selection

The selection of the dimensionality  $M$  of the transformed feature vectors  $\mathbf{s}_M$  is based on the assessment of the information conveyed by each one of its components, that is, the information conveyed along each axis of the coordinate system. Toward this end, the logarithmic entropy function<sup>17</sup> of the transformed feature vectors  $\mathbf{s} \in \mathbb{R}^N$ , defined as

$$H_R(\mathbf{s}) \triangleq - \sum_{j=1}^N v_j \log_2 v_j \quad (14)$$

with  $v_j$  defined as

$$v_j \triangleq \text{Var}[s_j] / \sum_{i=1}^N \text{Var}[s_i] \quad (j = 1, 2, \dots, N) \quad (15)$$

is used as a measure of the information conveyed by  $\mathbf{s}$ . Evidently,  $H_R(\mathbf{s})$  represents a decomposition of the information along each axis of the coordinate system  $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N\}$ .

The loss of information caused by truncating the transformed feature vector to dimension  $M$  ( $M < N$ ) then may be quantified by the difference between the logarithmic entropy of the original  $N$ -dimensional vector  $\mathbf{s}$  and that of its  $M$ -dimensional truncated counterpart  $\mathbf{s}_M$ . The smaller is the difference, the smaller is the loss of information, and thus the incurred approximation error. The truncation dimension  $M$  then may be selected on the basis of the quantity

$$\tilde{H}_R(\mathbf{s}_M, \mathbf{s}) \triangleq \frac{H_R(\mathbf{s}_M)}{H_R(\mathbf{s})} = \frac{\sum_{j=1}^M v_j \log_2 v_j}{\sum_{j=1}^N v_j \log_2 v_j} \in (0, 1] \quad (16)$$

which represents the fraction of the information maintained in the truncated vectors  $\mathbf{s}_M$ . Indeed, selecting an appropriate threshold value  $\delta$  (generally close to unity),  $M$  may be selected as the minimum truncation dimension for which the information maintained is greater than  $\delta$ ; that is,

$$M = \min_{\mu} \left\{ \mu \in [1, N] \mid \delta \leq \tilde{H}_R(\mathbf{s}_\mu, \mathbf{s}) = \frac{\sum_{j=1}^\mu v_j \log_2 v_j}{\sum_{j=1}^N v_j \log_2 v_j} \right\} \quad (17)$$

Because this study deals with feature vectors corresponding to the nominal (unfailed) system as well as to the system in various fault modes (with various fault magnitudes in each case),  $M$  may be selected as the maximum of the minimally required, in each case, truncation dimensions; that is,

$$M = \max_l \{M_l\} = \max_l \left\{ \min_{\mu_l} \left\{ \mu_l \in [1, N] \mid \delta \leq \tilde{H}_R(s_{\mu_l}, s) \right. \right. \\ \left. \left. = \frac{\sum_{j=1}^{\mu_l} v_j \log_2 v_j}{\sum_{j=1}^N v_j \log_2 v_j} \right\} \right\} \quad (l = 1, 2, \dots, p \cdot N_F) \quad (18)$$

with  $p$  representing the number of faults per fault mode used in the training stage and  $N_F$  the total number of fault modes considered.

### C. Fault Identification Approach

The fault identification approach consists of the following stages:

*Stage 1: Feature space selection and fault mode subspace construction (training stage, simulated data).* Based on data obtained from a detailed simulation model, the structure and interval parameter estimates of a partial and reduced-order system model are obtained through suitable identification techniques.<sup>13,14</sup> Let  $\theta$  denote the original  $N$ -dimensional feature vector composed of selected model parameters and characterized by mean  $\mu_\theta$  and covariance  $P_\theta$ .

Additional experiments, in which  $p$  faults of various magnitudes are for each one of the  $N_F$  fault modes injected into the detailed simulation model, are performed, and the corresponding interval estimates of  $\theta^{ij}$  computed [ $i_j$  refers to the  $j$ th fault ( $j = 1, \dots, p$ ) within the  $i$ th fault mode ( $i = 1, \dots, N_F$ )].

The feature vector estimates subsequently are transformed according to expressions (6) and (7), to produce the vectors  $s^{ij}$  along with their associated covariances  $P_{s^{ij}}$ . The reduced feature vector dimensionality  $M$  is selected according to Eq. (18), and the interval estimate of the reduced feature vector  $s_M^{ij}$  is determined by using Eqs. (11).

A  $\rho$ -dimensional stochastic feature space then is defined as the space spanned by the first and, potentially, the diagonal elements (variances) of the second-order moments of the reduced feature vectors  $s_M$ . Let these selected quantities compose the  $\rho$ -dimensional vector  $\theta_K$ .

The fault mode subspace representations are constructed by using linear  $(\rho - 1)$ -dimensional hyperplane approximations. The mathematical form of the  $i$ th fault mode hyperplane is

$$g^i(\theta_K) = \theta_{K_1} + \omega_1^i \theta_{K_2} + \dots + \omega_{\rho-1}^i \theta_{K_\rho} - \omega_\rho^i = 0 \\ \Rightarrow (\bar{\omega}^i)^T \theta_K - \omega_\rho^i = \theta_{K_1} + \bar{\theta}_K^T \cdot \omega^i = 0 \quad (19)$$

with  $\omega_l^i$  denoting the hyperplane's  $l$ th coefficient, and

$$\omega^i \triangleq [\omega_1^i \quad \omega_2^i \quad \dots \quad \omega_\rho^i]^T, \quad \bar{\omega}^i \triangleq [1 \quad \omega_1^i \quad \dots \quad \omega_{\rho-1}^i]^T \quad (20)$$

$$\theta_K \triangleq [\theta_{K_1} \quad \theta_{K_2} \quad \dots \quad \theta_{K_\rho}]^T, \quad \bar{\theta}_K \triangleq [\theta_{K_2} \quad \dots \quad \theta_{K_\rho} - 1]^T \quad (21)$$

Given the  $i$ th ( $i = 1, \dots, N_F$ ) fault mode estimates  $\theta_K^{ij}$  ( $j = 1, \dots, p$ ) with  $p > \rho$ , the corresponding hyperplane representation is estimated through linear regression<sup>15</sup> based on the model

$$\theta_{K_1}^{ij} + (\bar{\theta}_K^{ij})^T \cdot \omega^i = \epsilon^{ij} \quad (1 \leq j \leq p) \quad (22)$$

with  $\epsilon^{ij}$  denoting the  $j$ th regression error. The estimator is of the form

$$\hat{\omega}^i = -[(\bar{\theta}_K^{ij})^T \cdot \bar{\theta}_K^{ij}]^{-1} \cdot (\bar{\theta}_K^{ij})^T \cdot \theta_{K_1}^{ij} \quad (23)$$

with

$$\bar{\theta}_K^i \triangleq [\bar{\theta}_K^{i1} \quad \bar{\theta}_K^{i2} \quad \dots \quad \bar{\theta}_K^{ip}]^T \in \mathbb{R}^{p \times \rho} \quad (24)$$

and

$$\theta_{K_1}^i \triangleq [\theta_{K_1}^{i1} \quad \theta_{K_1}^{i2} \quad \dots \quad \theta_{K_1}^{ip}]^T \in \mathbb{R}^{p \times 1} \quad (25)$$

*Stage 2: Current feature vector estimation (experimental data).*

Once a fault is detected (see Ref. 12 for fault detection) based on periodically obtained system (operating) data, an interval estimate of the current (unknown fault) feature vector  $\theta^u$  is obtained and transformed according to Eq. (10) into  $s_M^u$ , from which  $\theta_K^u$  is formed [Eqs. (11)].

*Stage 3: Distance computations and fault identification (experimental data).* Appropriate distances between the current (unknown fault) point  $\theta_K^u$  and each fault mode hyperplane are subsequently computed. The distance between  $\theta_K^u$  and the  $i$ th mode hyperplane is obtained by optimizing the Lagrangian

$$L(\theta_K, \gamma) = D(\theta_K, \theta_K^u) + 2\gamma \cdot g^i(\theta_K) \quad (26)$$

with respect to  $\theta_K$  and  $\gamma$ . In this expression,  $2\gamma$  represents the Lagrange multiplier,  $g^i(\theta_K)$  the  $i$ th mode hyperplane defined by Eq. (19), and  $D(\cdot, \cdot)$  an appropriate distance function (metric) of the form

$$D(\theta_K, \theta_K^u) \triangleq \|\theta_K - \theta_K^u\|^2 \triangleq (\theta_K - \theta_K^u)^T \cdot (\theta_K - \theta_K^u) \quad (27)$$

The form of  $\theta_K$  depends on the metric space selected. In the deterministic metric space case,  $\theta_K \triangleq \mu_{s_M}$ , implying that only the mean of the truncated feature vector is used. In the stochastic metric space case,  $\theta_K \triangleq [\mu_{s_M}^T, (\text{col diag } P_{s_M})^T]^T$ , implying that both the mean and the diagonal elements of the truncated feature vector covariance are used (col diag  $P$  denotes the vector consisting of the diagonal elements of the square matrix  $P$ ).

The differentiation of the Lagrangian  $L(\theta_K, \gamma)$  [with  $D(\theta_K, \theta_K^u)$  defined by Eq. (27)] with respect to  $\theta_K$  and  $\gamma$  leads to the  $(\rho + 1)$  linear equations

$$\begin{bmatrix} I & \bar{\omega}^i \\ (\bar{\omega}^i)^T & 0 \end{bmatrix} \begin{bmatrix} \theta_K \\ \gamma \end{bmatrix} = \begin{bmatrix} \theta_K^u \\ \omega_\rho^i \end{bmatrix} \quad (28)$$

with  $I$  representing the  $\rho \times \rho$  identity matrix. These equations uniquely determine the projection  $\theta_K$  of  $\theta_K^u$  on the  $i$ th fault mode hyperplane and the corresponding  $\gamma$ . The distance between  $\theta_K^u$  and the hyperplane is computed by substituting the obtained  $\theta_K$  back into Eq. (27).

The current fault finally is identified as belonging to the  $i^*$ th fault mode if its distance from that mode's hyperplane is minimal; that is,

$$i^* = \text{index} \min_{i \in [1, N_F]} \left\{ \min_{\theta_K \in G^i} D(\theta_K, \theta_K^u) \right\} \quad (29)$$

with  $G^i \triangleq \{\theta_K \mid g^i(\theta_K) = 0\}$  and  $i = 1, 2, \dots, N_F$ . A measure of the fault magnitude may be obtained as  $D^{1/2}(\theta_K^o, \theta_K^u)$ , with  $\theta_K^o$  designating the vector corresponding to the nominal (unfailed) system.

## IV. Fault Identification Test Cases

The performance characteristics of the information-based reduced-dimensionality geometric approach are examined in connection with fault identification in two structural systems<sup>12</sup>: 1) a laboratory-scale beam and 2) a finite element model of a planar truss structure.

### A. Fault Identification in a Simply Supported Beam

#### 1. Experimental Setup and Preliminary Procedures

The beam (Fig. 1) is based on four rigid supports, dividing it into three spans, as well as on two additional flexible supports: one stationary, referred to as an auxiliary spring, and one movable that is at the midpoint of the central span in the nominal (unfailed) system case and is referred to simply as a spring. The considered faults correspond to deviations of the beam local stiffness characteristics (local changes in the modulus of elasticity) realized by transporting the movable support (spring) to any desired location in any span.

The objective of the experiments is fault identification, that is, the determination of the span in which a particular fault occurs, based

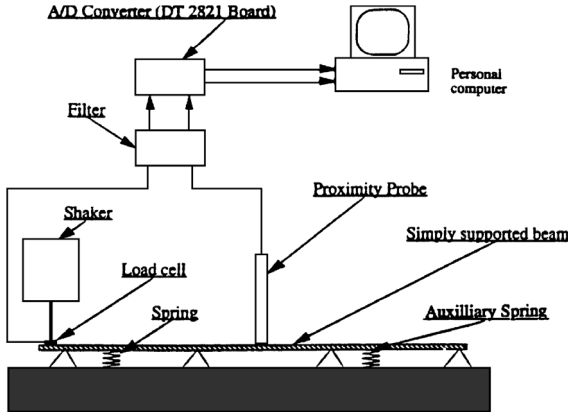


Fig. 1 Schematic diagram of simply supported beam experimental setup.<sup>12</sup>

on vibration test data obtained from a single pair of measurement locations. Three fault modes ( $N_F = 3$ ), denoted as  $F^1$ ,  $F^2$ , and  $F^3$  and corresponding to changes in the local stiffness characteristics at a point lying, respectively, anywhere in the first, second, and third span of the beam, are considered.

Fault identification is based on measurements of the exerted force and resulting vibration displacement at the midpoint of the central span, as obtained by a load cell and a proximity probe, respectively (Fig. 1). The two signals are driven through analog antialias filters (cutoff frequency at 400 Hz) and subsequently are digitized at 1 kHz.

Based on 1500-sample-long data records, the system dynamics are modeled by an estimated stochastic ARMAX (6, 5, 3) (autoregressive moving average with exogenous input) model of the form<sup>12</sup>

$$\sum_{i=0}^6 a_i \cdot y[t-i] = \sum_{i=1}^5 b_i \cdot F[t-i] + \sum_{i=0}^3 c_i \cdot w[t-i]$$

with  $a_0 \equiv c_0 \equiv 1$ ,  $y[t]$  representing the vibration displacement,  $F[t]$  the exerted force signal,  $w[t]$  a zero-mean and uncorrelated noise-generating stochastic sequence, and  $a_i$ ,  $b_i$ ,  $c_i$  the  $i$ th autoregressive (AR), exogenous (X), and moving average (MA) parameter, respectively. The vibration displacement  $y[t]$  and exerted force  $F[t]$  signals are measured, whereas the noise-generating stochastic sequence  $w[t]$  is not. The latter, along with the AR, X, and MA parameters and their covariance matrix, are estimated through appropriate stochastic estimation techniques (the interested reader should see Refs. 13 and 14 for procedures and details).

The foregoing ARMAX model attempts to account for both the structural and noise dynamics, although (for purposes of model simplicity) only three of the four structural modes present in the 0- to 400-Hz frequency range are represented. [As already mentioned, one of the strengths of the method is its ability to operate on reduced-order system representations, a feature becoming quite important when dealing with structures characterized by large numbers of degrees of freedom (DOF).] The original feature vector  $\theta$  consists of the model's autoregressive (AR) and exogenous (X) parameters (with dimensionality of  $N = 11$ ); that is,

$$\theta = [a_1 \quad \dots \quad a_6 \quad \vdots \quad b_1 \quad \dots \quad b_5]^T$$

Interval estimates of  $\theta$ , obtained via identification techniques operating upon data corresponding to faults realized at various points of each span during training, are transformed according to Eqs. (6) and (7), with the covariance matrix  $P$  selected as either  $P_{\theta^o}$  or  $P_{\theta^{av}}$  (Sec. III.A). Transformed feature vector dimensionality selection is based on the logarithmic entropy function, a normalized version of which is compared with the transformed feature vector dimension, for the various feature vectors, in Fig. 2 ( $P = P_{\theta^{av}}$ ). Applying the criterion of expression (18) with  $\delta = 0.970$ , a reduced feature vector  $s_M$  of dimensionality  $M = 9$  is selected. Following the selection of  $\theta_K$  as

$$\theta_K = [\mu_{s_1} \quad \dots \quad \mu_{s_M} \quad \vdots \quad \sigma_{s_1}^2 \quad \dots \quad \sigma_{s_M}^2]^T$$

Table 1 Fault identification results for simply supported beam<sup>a</sup>

Faults	Test case	Stochastic distance from hyperplane <sup>b</sup>		
		$F^1$	$F^2$	$F^3$
$F^1$ (span 1)	1	<b>2.7733e-01</b>	4.4287e-01	1.9341e-00
	2	<b>1.1813e-01</b>	1.4202e-00	1.5859e-00
	3	<b>3.3341e-01</b>	1.2078e-00	1.6386e-00
	4	<b>5.5534e-01</b>	7.6988e-01	1.9670e-00
	5	<b>1.1079e-00</b>	1.9402e-00	1.8091e-00
	6	<b>2.7344e-01</b>	3.8364e-01	2.0165e-00
$F^2$ (span 2)	7	3.5566e-00	<b>3.4436e-01</b>	1.6562e-00
	8	2.0700e-00	<b>1.0308e-01</b>	4.2317e-01
	9	3.8165e-00	<b>5.6934e-02</b>	4.1950e-01
	10	2.3498e-00	<b>7.5420e-02</b>	5.6329e-01
	11	4.4883e-00	<b>8.6493e-02</b>	4.2183e-01
	12	2.2013e-00	<b>1.0751e-02</b>	5.1501e-01
$F^3$ (span 3)	13	2.5372e-00	7.7870e-02	<b>3.3583e-02</b>
	14	1.5444e-00	<b>6.8563e-02*</b>	7.6861e-02
	15	1.2513e-00	8.0988e-02	<b>6.5441e-02</b>
	16	1.0696e-00	4.2530e-01	<b>5.2299e-02</b>
	17	4.1148e-01	<b>4.2383e-02*</b>	7.4034e-02
	18	2.6294e-00	4.7392e-01	<b>1.6920e-01</b>

<sup>a</sup>Information-based reduced-dimensionality geometric approach using the stochastic metric space.

<sup>b</sup>Minimal distance in each case is indicated in boldface; misclassification errors are marked with an asterisk.

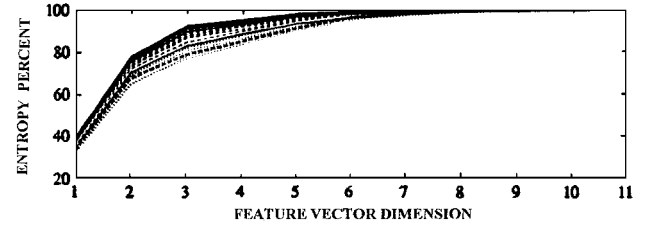


Fig. 2 Normalized logarithmic entropy of transformed feature vectors vs vector dimension (simply supported beam).

(dimensionality  $\rho = 18$ ) in the stochastic metric space case, the  $F^1$ ,  $F^2$ , and  $F^3$  fault mode hyperplanes are constructed according to the procedure of Sec. III.C. The  $i$ th fault mode hyperplane is thus of the form

$$g^i(\theta_K) = \theta_{K_1} + \omega_1^i \theta_{K_2} + \dots + \omega_{17}^i \theta_{K_{18}} - \omega_{18}^i = 0$$

Selected intersections of the first two hyperplanes ( $F^1$ ,  $F^2$ ) are depicted in Fig. 3 (the crosses indicate faults used in training), from which the linearity (hyperplane) approximation seems to work adequately for fault mode representation.

## 2. Results and Discussion

Eighteen test cases, the first six (1–6) corresponding to faults belonging to fault mode  $F^1$  (spring at  $kL/24$  with  $k = 1, 2, 3, 5, 6, 7$  and  $L$  denoting the length of the beam between the left and right rigid supports), the next six (7–12) corresponding to faults belonging to fault mode  $F^2$  (spring at  $kL/24$  with  $k = 9, 10, 11, 13, 14, 15$ ), and the last six (13–18) corresponding to faults belonging to fault mode  $F^3$  (spring at  $kL/24$  with  $k = 17, 18, 19, 21, 22, 23$ ), are considered. Note that none of these fault locations were used in the hyperplane construction (training) stage.

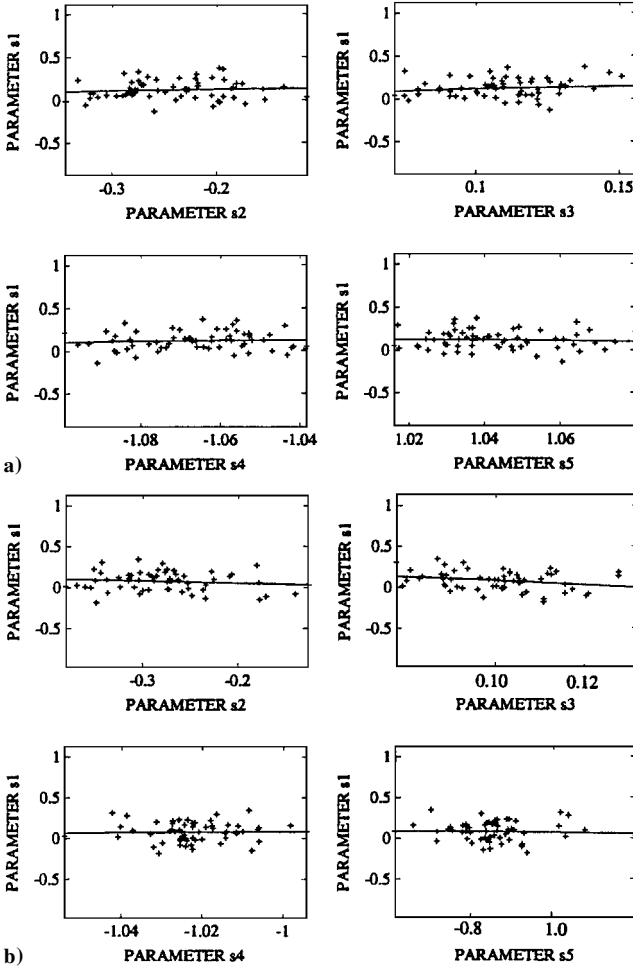
Fault identification results with the reduced-dimensionality geometric approach, using the transformed stochastic metric space, are presented in Table 1. In each test case the distances of the current (unknown fault) point to the fault mode hyperplanes are presented. Despite the difficulties of this particular setup and the use of a single transfer function model, the results are very satisfactory, with only two misclassification errors (test cases 14 and 17) encountered (compared to one in the original geometric approach<sup>12</sup>). As in the original case, a deterioration is observed when the deterministic metric space is used; in that case, four misclassification errors are encountered (compared to five in the original geometric approach<sup>12</sup>). This underscores the importance of using the stochastic metric space

**Table 2** Fault identification results for planar truss structure<sup>a</sup>

Faults	Test case	Stochastic distance from hyperplane <sup>b</sup>			
		$F^1$	$F^2$	$F^3$	$F^4$
$F^1$ (element 5)	1	<b>1.7926e-03</b>	1.3414e+03	1.9375e+03	2.8655e+01
	2	<b>2.3355e-02</b>	7.3055e+02	1.0528e+03	1.6189e+01
	3	<b>2.1655e-03</b>	3.4382e+02	4.9451e+02	6.8569e+00
	4	<b>4.8316e-03</b>	7.3410e+01	1.0505e+02	8.3727e-01
$F^2$ (element 6)	5	3.4969e+01	<b>2.4534e-02</b>	2.3671e+00	1.0011e+02
	6	7.7075e+01	<b>1.3862e+00</b>	8.8906e+00	1.1964e+02
	7	3.7926e+00	<b>9.5782e-02</b>	1.8827e+00	4.5451e+01
	8	8.0461e-01	<b>4.3134e-02</b>	5.0313e-02	6.0540e+00
$F^3$ (element 7)	9	3.9609e+01	1.6177e+00	<b>3.2396e-01</b>	1.2087e+02
	10	6.8187e+01	2.8488e+00	<b>1.1327e-01</b>	6.3781e+01
	11	1.9624e-01	2.8949e-01	<b>1.9723e-02</b>	2.6046e+01
	12	8.2346e-01	<b>2.6831e-02*</b>	3.3711e-02	5.9441e+00
$F^4$ (element 8)	13	7.6453e+00	1.0286e+02	3.4041e+02	<b>2.8001e+00</b>
	14	2.2538e+01	3.0629e+02	4.6158e+02	<b>2.2996e+00</b>
	15	3.8171e+00	1.5643e+02	2.2471e+02	<b>1.8567e+00</b>
	16	1.7780e+00	4.5139e+01	6.4849e+01	<b>2.5259e-01</b>

<sup>a</sup>Information-based reduced-dimensionality geometric approach using the stochastic metric space.

<sup>b</sup>Minimal distance in each case is indicated in boldface; misclassification errors are marked with an asterisk.



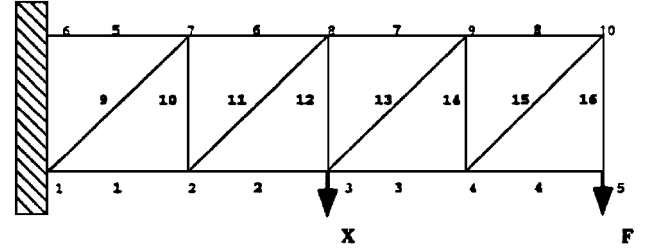
**Fig. 3** Selected intersections of a)  $F^1$  and b)  $F^2$  hyperplanes (simply supported beam).

in conjunction with systems characterized by nonnegligible random effects.

## B. Fault Identification in a Truss Structure

### 1. Structure and Preliminary Procedures

The planar truss structure consists of 16 components (steel bars) connected at 10 joints (Fig. 4). The length of elements 1–8, 10, 12, 14, and 16 is  $L = 5 \times 10^{-1}$  m, whereas that of elements 9, 11, 13, and 15 is  $L = 5\sqrt{2} \times 10^{-1}$  m. Their cross-sectional area is  $A = 4 \times 10^{-4}$  m<sup>2</sup>, their specific mass is  $\rho = 7860$  kg/m<sup>3</sup>, and their



**Fig. 4** Schematic diagram of the planar truss structure.<sup>12</sup>

modulus of elasticity is  $E = 200$  GPa. The structure is modeled by a 16-DOF finite element model. The structural damping considered is of the proportional type, with the damping matrix being 2% of that of the stiffness. The vibration response is computed by integrating the equations of motion using the Newmark integration method<sup>18</sup> (integration step of 0.001 s).

The faults considered are deviations of the local stiffness characteristics (modulus of elasticity) of the various structural components (bars). The objective of the experiments is fault identification, that is, the determination of the particular component (bar) in which a fault occurs based on vibration test data. Four fault modes are considered: Fault mode  $F^1$  consists of the faults of all possible magnitudes occurring in element 5,  $F^2$  of those occurring in element 6,  $F^3$  of those occurring in element 7, and  $F^4$  of those occurring in element 8.

As in the beam case, fault identification is based on measurement of the stochastic (zero-mean and uncorrelated) force excitation applied at joint 5 and the resulting vibration displacement at joint 3 (both along the vertical  $x$  direction).

The use of model order determination and parameter estimation techniques<sup>13,14</sup> in conjunction with the ARMAX model form and 1500-sample-long data records, leads to an ARX (6, 5) representation<sup>12</sup>:

$$\sum_{i=0}^6 a_i \cdot y[t-i] = \sum_{i=1}^5 b_i \cdot F[t-i] + w[t]$$

with  $a_0 \equiv 1$ . The original feature vector  $\theta$  consists of the model's autoregressive (AR) and exogenous (X) parameters, with its dimensionality being  $N = 11$ .

Interval estimates of  $\theta$ , obtained via identification techniques operating upon data corresponding to faults of various magnitudes for each bar, are transformed according to Eqs. (6) and (7). Transformed feature vector dimensionality selection is based on the logarithmic entropy function, a normalized version of which, for the various feature vectors, is presented in Fig. 5 ( $P = P_{\theta_{av}}$ ). Applying the criterion of expression (18) with  $\delta = 0.970$ , a reduced feature vector

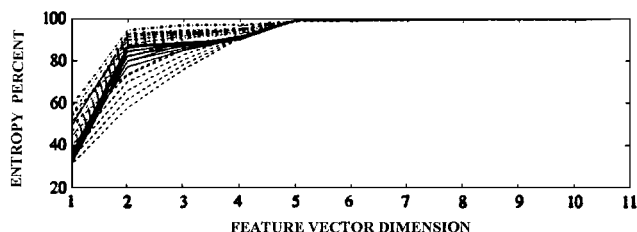


Fig. 5 Normalized logarithmic entropy of the transformed feature vectors versus vector dimension (planar truss structure).

dimensionality of  $M = 8$  is selected. Following the selection of  $\theta_K$  (of dimensionality  $\rho = 16$  in the stochastic metric space), the  $F^1$ - $F^4$  fault mode hyperplanes are constructed.

## 2. Results and Discussion

Sixteen test cases, divided into four groups, with each group corresponding to each one of the considered fault modes, are examined. As in the beam experiment, none of the considered fault magnitudes was used in the hyperplane construction stage.

Fault identification results with the reduced-dimensionality geometric approach using the transformed stochastic metric space, are presented in Table 2. In each test case the distances of the current (unknown fault) point to the fault mode hyperplanes are presented. The results are very satisfactory, with only one misclassification error (test case 12) encountered, a performance that is exactly equivalent to that of the original geometric approach.<sup>12</sup>

## V. Conclusions

An information-based reduced-dimensionality formulation of the geometric fault identification approach is introduced. According to this formulation, the feature vector is transformed into a suitable coordinate system, within which information compression, and thus feature vector truncation, may be best achieved. Feature vector dimensionality selection is based on bounding of the information loss, expressed in terms of logarithmic entropy, below a certain threshold. The information-based reduced-dimensionality formulation offers a formal and systematic procedure for the proper selection of a minimal dimensionality feature vector and a corresponding feature space, thus achieving two important goals: 1) substantial simplification of the fault identification procedure, and 2) potentially improved fault identification performance over arbitrarily truncated feature vectors.

The performance characteristics of the approach were demonstrated via fault identification in a laboratory-scalesimply supported beam and a finite element model of a planar truss structure. In both cases the reduced-dimensionality ( $M = 9$  and  $M = 8$  for the beam and truss, respectively, as opposed to original dimensionalities of  $N = 11$ ) formulation performed essentially equivalently to the original, full-dimensionality version.

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